

Real Time Forecasts through an Earthquake Clustering Model Constrained by the Rate-and-State Constitutive Law: Comparison with a Purely Stochastic ETAS Model

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INTRODUCTION

We propose an earthquake clustering model based on the popular concept of epidemic models. In these models every earthquake can be regarded as both triggered by previous events and as a potential triggering event for subsequent earthquakes (Ogata 1988, 1998; Ogata and Zhuang 2006 and reference therein; Console and Murru 2001; Console *et al.* 2003; Console *et al.* 2006a, 2006b; Helmstetter and Sornette 2002a, 2002b, 2003 for reviews; and Vere-Jones 2006 for review on the use of stochastic models for earthquake occurrence). The occurrence-rate density at any time and geographical location is computed by the contribution of every previous event using a kernel function that takes into proper account: (a) the magnitude of the triggering earthquake, (b) the spatial distance from the triggering event, and (c) the time interval between the triggering event and the instant considered for the computation. The magnitude distribution adopted here is the Gutenberg-Richter law (Gutenberg and Richter 1944). The above-mentioned criteria are implemented through the introduction of the rate-and-state constitutive law in a previously existing epidemic algorithm. The validity of the model can be tested in an exercise of real-time forecast.

Stochastic short-term models describing the phenomenon of earthquake clustering are achieving increasing success in the seismological community (*e.g.*, Helmstetter *et al.* 2006). Progress is also being made with models that link stress changes to seismicity rate changes using the Dieterich rate-and-state model (Ruina 1983; Dieterich 1986, 1992, 1994; Console *et al.* 2006a). In any such model, earthquakes are regarded as the realization of a point process modeled by a generalized Poisson distribution. Each event is characterized by its location-time-magnitude parameters (x, y, z, t, m) . Depth is not used in our analysis considering its limited range (<30 km) in comparison with the horizontal scale. Both the purely stochastic and the physically constrained models have in common the concept that all earth-

quakes have identical roles in the triggering process. In this process every earthquake is a potential main shock, with its own aftershock sequence decaying according to the modified Omori law or the rate-and-state model (for the stochastic and physical-stochastic models, respectively) and with magnitude distribution following the Gutenberg-Richter law (Gutenberg and Richter 1944), with a constant b -value. In both methods the occurrence-rate density of seismic events is the superposition of a time independent (Poissonian) component and the seismic activity induced by previous earthquakes. Moreover, for both models the spatial distribution of induced events is described by an isotropic function around the epicenter of every previous event including the dependence on the magnitude of the previous events. This type of formulation of occurrence-rate density allows us to estimate the likelihood function of an earthquake catalog and to assess the maximum likelihood parameters of the hypothesis. Another possibility given by such formulation is the computation of the likelihood ratio (performance factor) of two models based on a given catalog as a measure of their relative information value. This allows a systematic approach to the improvement of the models through formal testing of the value of proposed innovations. The application of such models to seismicity in Japan showed that the performance of the physically constrained stochastic model, incorporating the concept of the rate-and-state theory with two free parameters only, is comparable to that of the purely stochastic model (Console *et al.* 2006a). To test the respective performance of the models in the context of moderate seismicity, both models have been fitted to the early part of the California earthquake catalog (1984.01–1992.09) and then tested on the most recent part of the data set (1992.10–2004.12).

SHORT-TERM FORECASTING MODELS BASED ON EARTHQUAKE CLUSTERING

Here we give a brief outline of epidemic models incorporating the short-term clustering properties of earthquakes.

Assuming the validity of the Gutenberg-Richter law, the occurrence-rate density of seismic events triggered by previous earthquakes can be described as:

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$$\lambda(x, y, t, m) = f_r \cdot \lambda_0(x, y, m) + \sum_{i=1}^N H(t - t_i) \cdot \lambda_i(x, y, t, m), \quad (1)$$

where f_r is a factor called the “failure rate” (*i.e.*, the ratio between the expected number of independent events and the total number of events); $\lambda_0(x, y, m)$ represents the time-invariant background seismicity, obtained from the analyzed catalog by means of a smoothing algorithm (Console and Murru 2001); t_i is the occurrence time of the N earthquakes; $H(t)$ is the step function, and $\lambda_i(x, y, t, m)$ is the single contribution of the previous earthquakes. The first and the second terms on the right hand side of equation (1) represent the time-invariant “spontaneous” and the time-varying “induced” seismicity, respectively (Ogata 1998; Console and Murru 2001; Console *et al.* 2003; Console *et al.* 2006a, 2006b). The rate density corresponding to any earthquake is the sum of these two components.

Equation (1) describes a generic epidemic process that can be characterized by modeling the contribution of each previous earthquake, λ_i . In this way we talk about a “purely stochastic” versus a “physically constrained” epidemic model (Console *et al.* 2006a).

A PURELY STOCHASTIC EPIDEMIC MODEL FOR EARTHQUAKE CLUSTERING

Building up a purely stochastic model called the epidemic type aftershock sequence (ETAS) model we hypothesize that three terms, representing the time, magnitude, and space distribution respectively, specify the contribution of any previous earthquake (for $t > t_i$):

$$\lambda_i(x, y, t, m) = K \cdot b(t - t_i) \cdot \beta e^{-\beta(m - m_i)} \cdot f(x - x_i, y - y_i) \quad (2)$$

where K is a constant and $\beta = b \ln(10)$. For the time dependence, we adopt a modified Omori law (Ogata 1983) for first and secondary aftershocks:

$$b(t) = (p - 1)c^{(p-1)}(t + c)^{-p} \quad (p > 1), \quad (3)$$

where c and p are characteristic parameters of the process, and the expression is normalized so that $\int_0^\infty b(t) dt = 1$.

For the magnitude distribution we assume the Gutenberg-Richter law is valid for all the earthquakes in the sample with a constant β . Finally, for the spatial distribution of triggered seismicity we modeled a function, normalized to 1, with circular symmetry around the source point (x_i, y_i) , given in polar coordinates as

$$f(r, \theta) = \frac{q-1}{\pi} \cdot \frac{d^{2(q-1)}}{(r^2 + d^2)^q} \quad (4)$$

where d and q are two free parameters and r is the distance from the point (x_i, y_i) . This model has five free parameters: K , two free parameters of the modified Omori formula (c and p), and two of the spatial distribution. For other details see Console *et al.* (2006).

A PHYSICALLY CONSTRAINED EPIDEMIC MODEL: APPLICATION OF THE RATE-STATE MODEL TO EARTHQUAKE CLUSTERING

We build up a new clustering model (Console *et al.* 2006a; Console and Catalli 2006) physically constrained by the rate-and-state constitutive law introduced by Ruina (1983) and Dieterich (1986, 1992, 1994). We call this the epidemic rate-strain (ERS) model. We merged in a single algorithm the classical concept of ETAS (the purely stochastic model) and the rate-state theory for the seismicity rate. The final model is stochastic, which allows the computation of the likelihood of a seismic catalog, but it also reflects at least to some extent the physics of earthquake processes.

According to Dieterich (1994) the rate $R(t)$ of earthquakes after a $\Delta\tau$ stress change at time $t = 0$ is given by

$$R(t) = \frac{R_0}{1 - \left[1 - \exp\left(\frac{-\Delta\tau}{A\sigma}\right) \right] \exp\left(\frac{-t}{t_a}\right)} \quad (5)$$

where R_0 is the previous background-rate density, and A , σ and t_a are physical parameters of the constitutive law. For all of our applications A and σ always appear as a multiplicative product, so we retain their product as only one independent parameter. It is also possible to show (Dieterich 1994; Console *et al.* 2006a) that t_a is equal to $A\sigma$ divided by the stressing rate of the seismic area, $\dot{\tau}$, and the latter is itself related to the background rate R_0 . Rather than computing the stress change $\Delta\tau$, caused at any point by any earthquake, for which it would be necessary to know the source parameters of all the earthquakes, we introduce a shortcut that allows the use of the most common catalog information: the origin time, the epicentral coordinates, and the magnitude. Empirically we hypothesize that the stress change produced by an earthquake is given by

$$\Delta\tau = \Delta\tau_0 \left(\frac{d_i^2}{r^2 + d_i^2} \right)^q \quad (6)$$

where r and q have the same meaning as in equation (4), $\Delta\tau_0$ is a free parameter representing the maximum shear stress produced by the fault at its epicenter and $d_i = d_0 e^{\alpha(m_i - m_0)}$, where d_0 is the radius of a circular fault of magnitude m_0 (for details, see Console *et al.* 2006a).

For numerical applications it is necessary to define the value of the various parameters. In this study we aim to reduce the number of free parameters as much as possible, so we arbitrarily fix the value of some parameters that can be reasonably estimated based on prior experience:

- q is fixed equal to 1.5, for consistency with the theory of elasticity when $r \rightarrow \infty$;
- b is estimated from the analyzed catalog;
- R_0 is obtained from the analyzed catalog by means of a smoothing algorithm (Console and Murru 2001).

We leave only $\Delta\tau_0$ and the product $A\sigma$ as free parameters of the new model, to be determined by a maximum-likelihood best fit.

TESTING THE MODELS WITH CALIFORNIA SEISMICITY

The procedure adopted in this study reflects those used by Console and Murru (2001), Console *et al.* (2003), and Console *et al.* (2006a) and previously introduced by Ogata (1998). It consists in searching for the maximum of the log-likelihood function of a realization of seismic events described by a catalog $\{x_j, y_j, t_j, m_j, j = 1, \dots, N\}$:

$$\ln L = \sum_{j=1}^N \ln \left[\lambda(x_j, y_j, t_j, m_j) V_0 \right] - \int \int \int \int \lambda(x, y, t, m) dx dy dt dm \quad (7)$$

where V_0 is a coefficient whose dimensions are equal to those of the inverse of the rate density $\lambda(x_j, y_j, t_j, m_j)$.

We are interested in assessing the predictive power of the models for earthquakes that will have an impact on the population. We thus consider the likelihood of the observed earthquakes under the respective models using only earthquakes with magnitude equal to or larger than $m_m = 4.0$ as target events. This is the same threshold considered for California in the Regional Earthquake Likelihood Models (RELM) test exercise (http://www.earthquake.ethz.ch/docs/presentations/talk_schorlemmer2005.pdf). In this way, the sum in the first member of the right side of equation (7) and the integral in the second member are computed only for earthquakes with magnitude $m \geq 4.0$, but the occurrence-rate density is computed taking into account the triggering effect of all the earthquakes exceeding magnitude m_0 .

The method outlined above has been implemented on the (undeclustered) California seismic catalog collected for the RELM test exercise (Schorlemmer *et al.* 2004) from 1984.01 to 2004.12, containing 14,823 earthquakes of magnitude equal to or larger than 3 with depth $b \leq 30$ km contained in the rectangle of 1,800 km by 1,800 km centered on the point of coordinates 36.5°N and -119°E, including the entire state of California. This selection reduces the data set to 14,699 events. The largest recorded magnitude is $m_{\max} = 7.3$. The maximum-likelihood b -value of the data set is 0.852 ± 0.007 with 95% error computed using the formula suggested by Shi and Bolt (1982). Figure 1 shows a map of the smoothed seismicity for the whole data set in terms of number of events with magnitude $m \geq 3.0$ in cells of 25 km².

The catalog has been divided in two parts: a learning period from 1984.01 to 1992.09 and a test period from 1992.10 to 2004.12. This choice has been guided by the criterion that each of the periods would include enough events to achieve a good reliability of the results. The first part of the catalog (learning data set), containing 7,346 events of magnitude $m \geq 3.0$ and 953 events of magnitude $m \geq 4.0$, has been used for the best fit of the free parameters characterizing models ETAS (purely stochastic) and ERS (physically constrained), respectively. The maximum-likelihood b -value of the learning data set is 0.859 ± 0.01 . The second part (testing data set), containing 7,352 events

Lower magnitude threshold of target events	4.0
$K(\text{days}^{\beta-1})$ Productivity coefficient in equation (2)	0.00288
c (days) Time constant in Omori law	0.00437
ρ Exponent in Omori law	1.146
d (km) Characteristic distance in the spatial distribution	1.57
f_r Fraction of spontaneous events	0.387
Maximum log-likelihood	13393.21

Lower magnitude threshold of target events	4.0
$\Delta\tau_0$ (MPa) Maximum shear stress at the epicenter	0.17
$A\sigma$ (MPa) Physical parameters of the constitutive law	0.0057
f_r Fraction of spontaneous events	0.288
Maximum log-likelihood	12857.06

of magnitude $m \geq 3.0$ and 940 events of magnitude $m \geq 4.0$, has been used for the evaluation of the likelihood for the two models in forward-retrospective way.

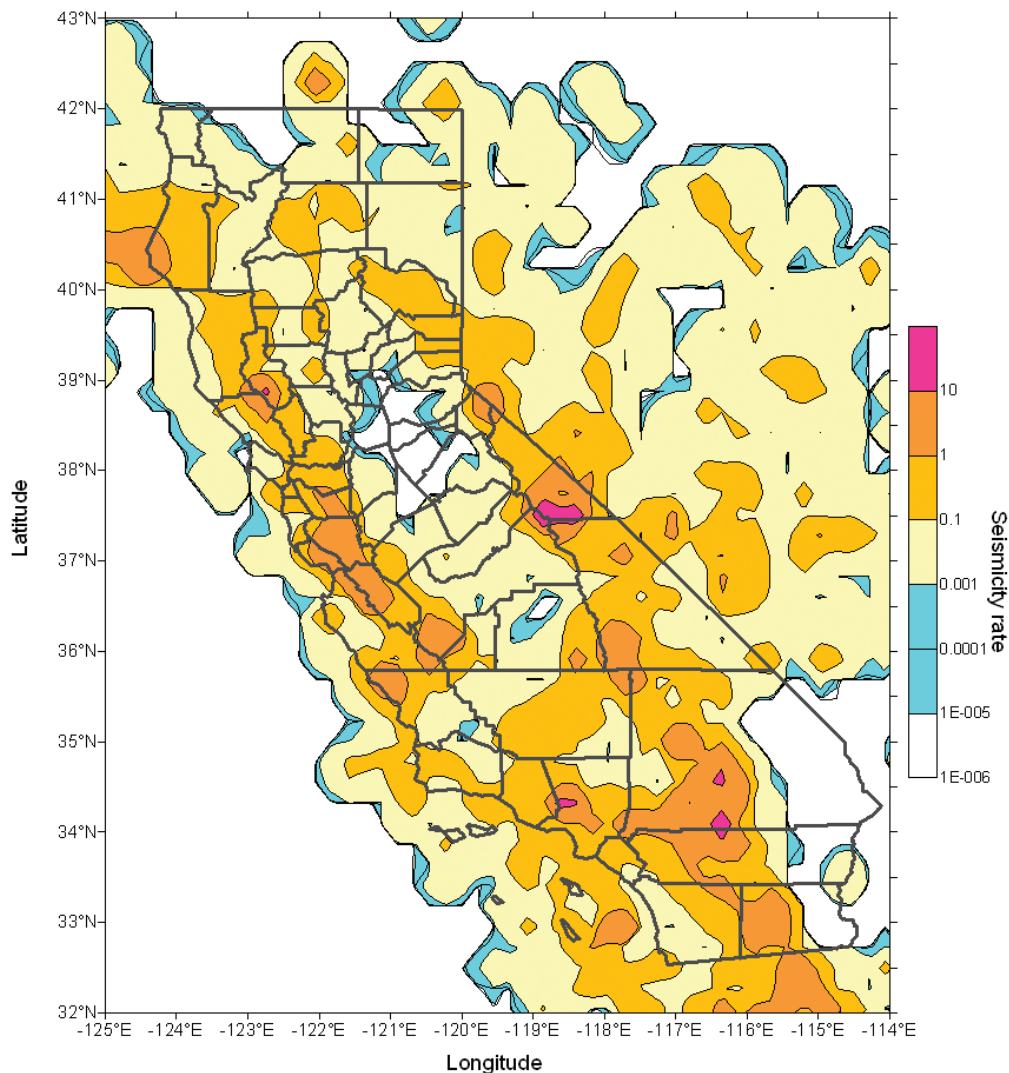
The space density of earthquakes of magnitude equal to or larger than m_0 , $\mu_0(x, y)$ has been computed for the learning data set by interpolation of a grid of 5 km by 5 km. The values at each node of the grid have been obtained by a smoothing algorithm over all the events of this data set with a correlation distance of 11 km, obtained for California by the method introduced in Console and Murru (2001).

As noted we fixed the value of q at 1.5 to limit the number of free parameters in the model. For the remaining four free parameters of model ETAS, the maximum-likelihood best fit has provided the results shown in table 1.

For the ERS model, having chosen $m_0 = 3.0$ (from which $M_0^* = 4.0 \times 10^{13}$ Nm and $d_0 = 190$ m have been computed), and $\alpha = \beta$, the best fit provides the results shown in table 2.

Note that the log-likelihood for model ERS is lower than that for model ETAS. The difference is of the order of 536, large enough to reject the physically constrained hypothesis in favor of the stochastic model according to the Akaike Information Criterion (Akaike 1977). Note also that the log-likelihood under a time-independent Poisson model on the same catalog is only 10435.50 (table 3), so as to allow rejecting this null hypothesis in favor of the ETAS and ERS models.

The test is carried out on a different and independent data set (1992.10–2004.12). In the test we run the algorithm for the likelihood computation without looking for a best fit, using



▲ **Figure 1.** Smoothed seismicity of the California region for the period 1984.01–2004.12, using 11 km as the value of the correlation distance. The color scale represents the number of earthquakes ($m \geq 3.0$ and $h \leq 30$ km) in an area of 25 km², over the whole time period spanned by the catalog.

the parameters obtained from the learning phase. The effective number of free parameters in the test is zero for all the models, so that the Akaike Information Criterion is not applicable. The results are shown in table 3.

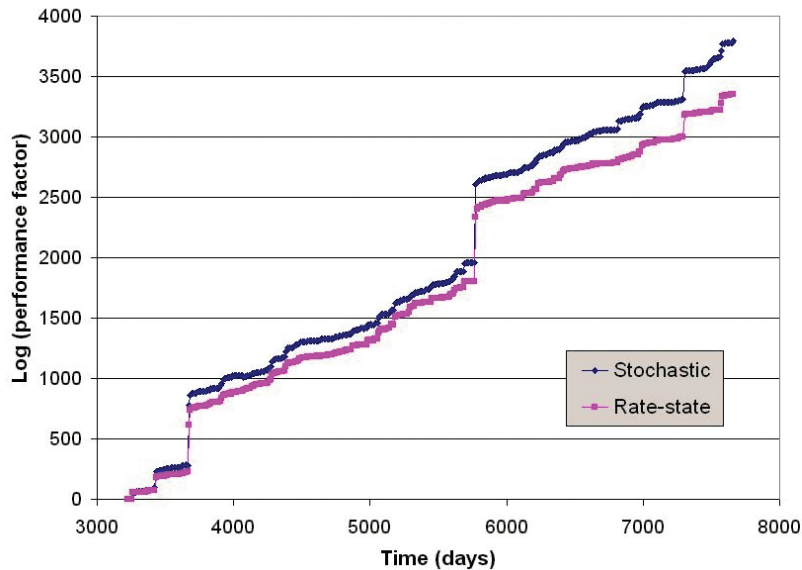
The ERS model yields likelihood larger than that of a simple time-independent Poisson model, but smaller than that of the ETAS model.

Figure 2 shows the temporal variation of the log-performance factor for both models ETAS and ERS with respect to the time-independent Poisson model for the test period. Each of the events with magnitude equal to or larger than 4.0 produces a sharp step in the performance factor. The step is positive when the occurrence rate expected by the epidemic model is larger than that expected by the Poisson model. Note the difference in the size of these steps between the two models considered. In general, the ETAS model achieves larger steps for all the earthquakes. Figure 2 shows the trend of the performance factor in the time periods between the target events. This trend

TABLE 3 Performance of Models in the Test with the 1992.10–2004.12 Data Set	
Model	Log-likelihood
Poisson time independent	10435.50
Purely stochastic model (ETAS)	12597.26
Physically constrained (ERS)	11717.30

is negative if the occurrence rate expected by the Poisson model for the whole target area is smaller than that expected by the epidemic model, as is generally the case.

In figures 3(A) and (B) and 4 (A) and (B) we show examples of how the ETAS and ERS models could be applied in real cases to display the spatial changes of the expected occurrence-rate density. The values of all free parameters used in the models are



▲ **Figure 2.** Evolution of the performance factor achieved by a pure stochastic model and an earthquake clustering model constrained by the rate-and-state constitutive law for a target magnitude threshold $m = 4.0$.

those obtained by the maximum log-likelihood best fit (tables 1 and 2) and the background seismicity is that obtained from the learning period 1984.01–1992.09.

The maps in figures 3 and 4 can then be considered real forecasts of the subsequent activity. The example refers to the M_w 7.1 Hector Mine earthquake of 16 October 1999 at 09:41 UTC. Figures 3(A) and 4(A) show the maps of the expected occurrence-rate density ($M \geq 4.0$) on 16 October 1999 at 00:00 UTC before the foreshock of magnitude 3.7, the epicenter of which is shown by a black star. The occurrence-rate density before this foreshock, ranging from 1×10^{-5} to 1×10^{-4} , is dominated by the spontaneous component of the seismicity. Figures 3(B) and 4(B) show the occurrence-rate density affected by the foreshock activity at 09:30 UTC on 16 October 1999, before the mainshock of M_w 7.1 (09:46 UTC), shown by the black star.

In summer 2006 the ETAS model was implemented in a test of real-time forecasting of Italian seismicity. The test algorithm automatically makes use of the seismic observations collected at the Monitoring Center of the Istituto Nazionale di Geofisica e Vulcanologia, which can be considered complete above a magnitude threshold of 2.4. The test area is $1,000 \text{ km} \times 1,200 \text{ km}$ wide and it is divided in square cells of $100 \text{ km} \times 100 \text{ km}$. For each cell the forecast is carried out over a time interval of 12 hours and 10 different magnitude thresholds. The verification procedure is based on the relative operating characteristic (ROC) diagram that has long been used for forecast verification in the atmospheric sciences (*e.g.*, in binary forecasting of tornadoes, El Niños). The ROC diagram considers the fraction of failures-to-predict and the fraction of false alarms (Holliday *et al.* 2005).

Like the ETAS model, the ERS model can also be tested for validity on real-time seismicity. In fact they both provide a time-dependent occurrence-rate density that can be integrated in appropriate space-time windows and used for the compu-

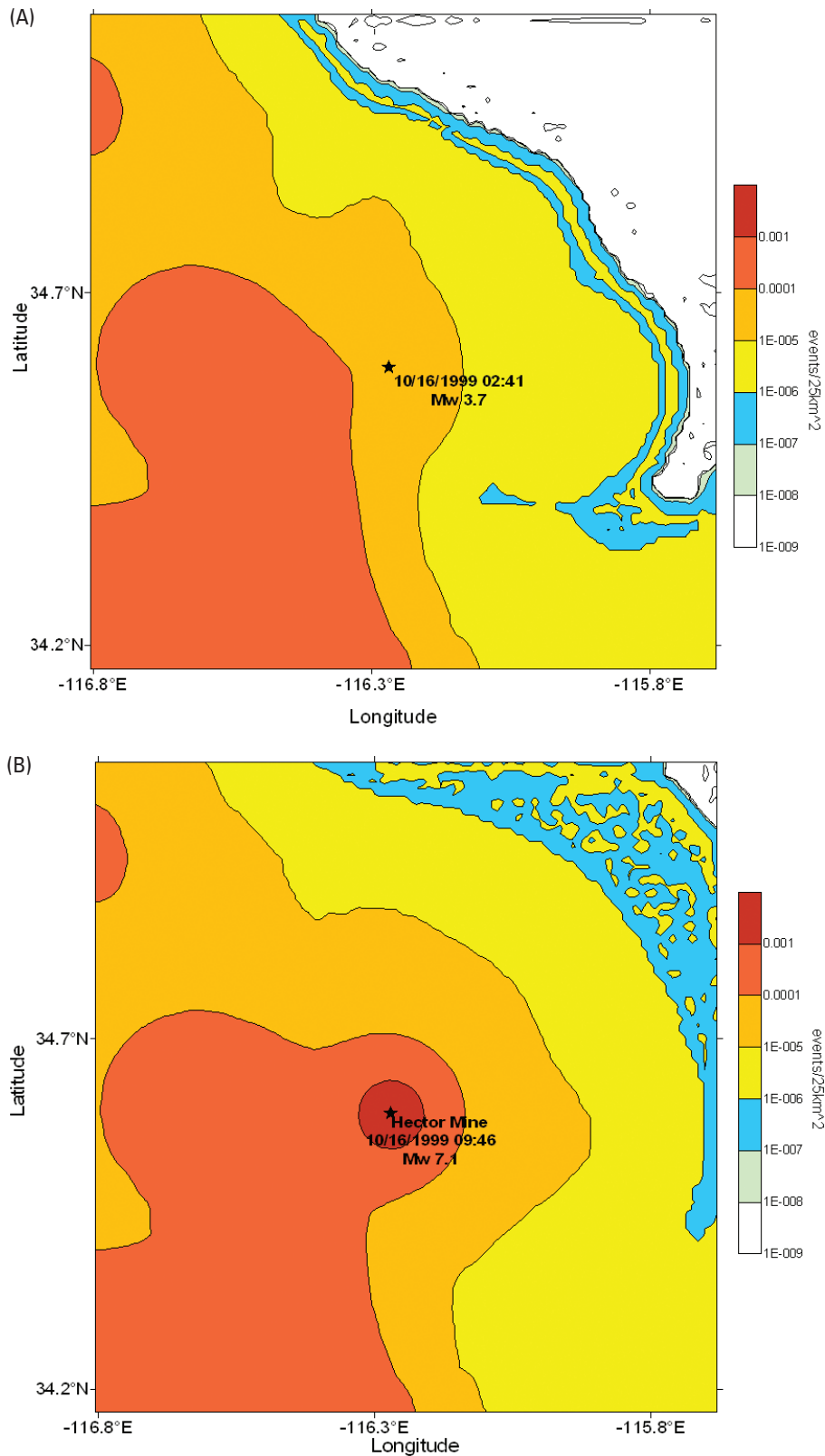
tation of the likelihood of the observations under the specific model. We consider the ERS model ready to take part in the RELM test exercise (Schorlemmer *et al.* 2004).

DISCUSSION AND CONCLUSIONS

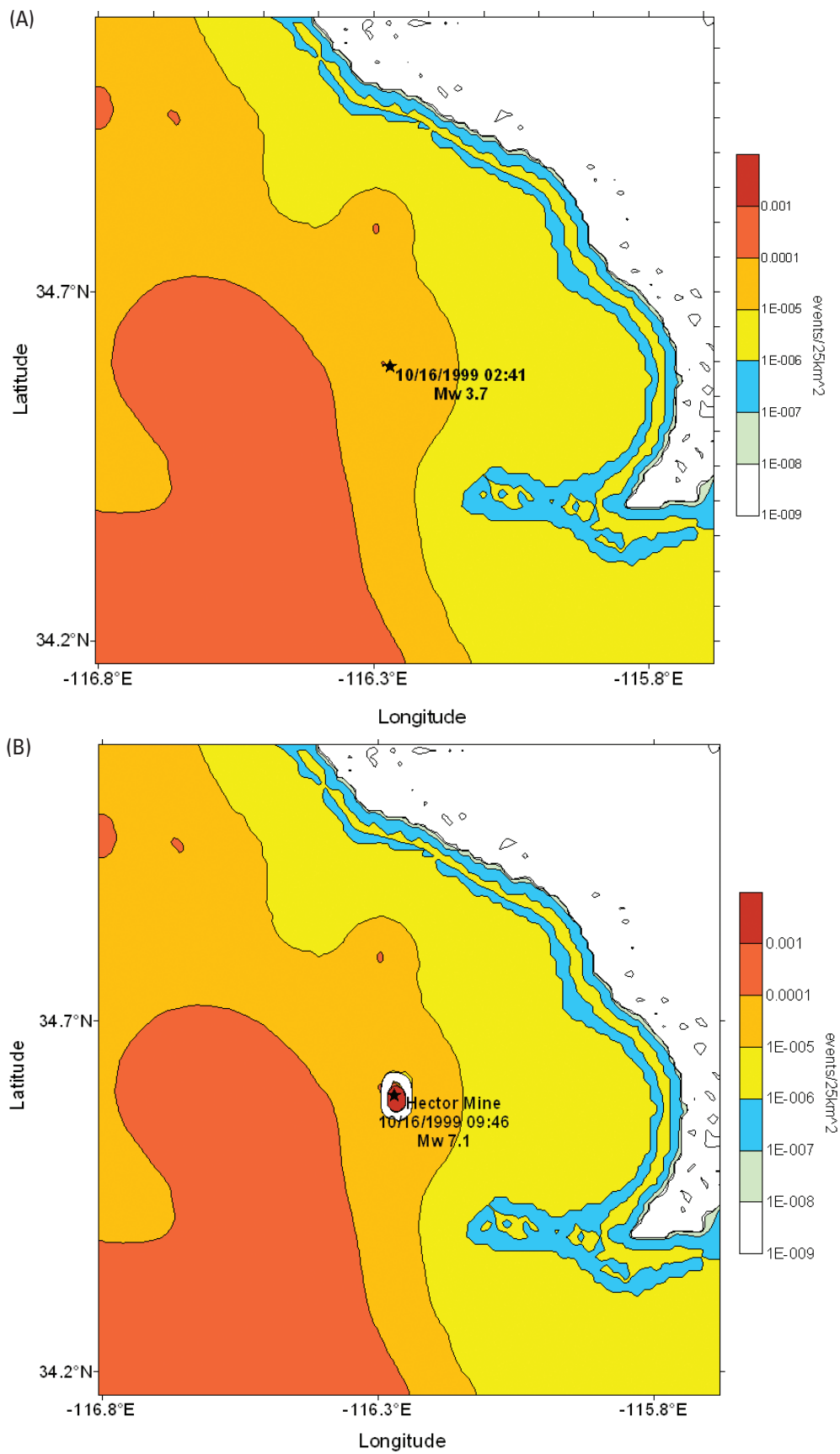
The strong difference in the spatial distribution modeled by the ETAS and ERS models may help explain the better performance of the former when applied to the seismicity of California. The ETAS model contains the d parameter, which can be adjusted to account for the location errors of the epicenters in the catalog, typically of the order of magnitude of several kilometers. In this respect model ERS is much more rigid, as the spatial distribution doesn't depend on any free parameter, but is constrained by the relation between magnitude and source size. Because the source size for magnitudes as small as $M = 3.0$ is smaller than 200 m (Kagan 2002), the much larger spread introduced by the location errors affects the likelihood of the catalog to a large extent. This circumstance also explains why the ERS model exhibited a better performance with the Japanese catalog when we considered only magnitudes larger than $M = 4.5$ (Console *et al.* 2006a).

Considering the physical implications of the values obtained for the parameters $\Delta\tau_0$ and $A\sigma$ in model ERS, we note that $\Delta\tau_0 = 0.17 \text{ MPa}$ (1.7 bars) is about 15 times smaller than the constant stress drop value, $\Delta\sigma$, assumed in the source model. We must also consider that $\Delta\tau_0$ is an average result of extremely complex real situations. These situations may include strong stress change variations over the fault area, from negative values to larger positive values, in correspondence with barriers.

Our result for $A\sigma$ (0.0057 MPa) is smaller than that (0.04 MPa) obtained by Toda and Stein (2003) in their study of aftershocks triggered by a couplet of earthquakes in Southern Kyushu. Nevertheless it appears consistent with the result



▲ **Figure 3.** (A) Modeled occurrence-rate density, $m \geq 4.0$, (events/day/25 km²) in a square area of 100 km × 100 km surrounding the epicentral area of the Hector Mine 16 October 1999 earthquake (7.1 M_w , 09:46 UTC), before its foreshock of 02:41 UTC (3.7 M_w). A black star shows the epicenter of the foreshock. The stochastic model of earthquake clustering (ETAS) has been applied for the computation of the occurrence-rate density, using the maximum likelihood parameters shown in table 1. (B) As in (A) on 16 October 1999 at 09:30 UTC, just before the mainshock.



▲ **Figure 4.** (A) and (B). As in figure 3 (A and B respectively) but considering the physical and stochastic model of earthquake clustering, ERS. The values of failure rate f_r , $\Delta\tau_0$ and $A\sigma$ used for the computation are those obtained by the maximum log-likelihood best fit (table 2).

obtained by Harris and Simpson (1998), who estimated for $A\sigma$ a range of acceptable values of 0.0012 MPa to 0.6 MPa in a study of earthquake interaction in the San Francisco Bay area. According to Dieterich (1995), who assumed values ranging from $A = 0.00001$ to $A = 0.007$ in his simulations, our result would lead to a normal stress ranging from 0.8 to 570 MPa in the seismogenic layer. The upper limit of this range is consistent with the lithostatic pressure existing in the seismogenic layer at a depth of 5–10 km.

A conversion of the background rate R_0 into the stress rate $\dot{\tau}$ for the California seismic catalog leads to an average stress rate $\dot{\tau} = 47.4$ Pa/day over the entire seismogenic area. Assuming a shear module of 3×10^4 MPa, this value roughly corresponds to a strain rate of 5.8×10^{-7} / year. Dividing $A\sigma = 0.0057$ MPa by $\dot{\tau} = 47.4$ Pa / day, we obtain $t_a = 120$ days, a reasonable value for the average duration of aftershock sequences according to the analysis of the catalog.

The application of the rate-and-state constitutive law reduces the number of free parameters of the model, giving at the same time a physical meaning to each of them. Regardless of the fairly poor performance of the physically constrained model of clustered seismicity in the present application, the study of this kind of model may provide insights into seismogenic processes.

The algorithm and the computer code by which the ERS model have been implemented make it suitable for real-time estimate of the likelihood of a set of seismic observations. These features make the model ready for the RELM test exercise. ☒

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